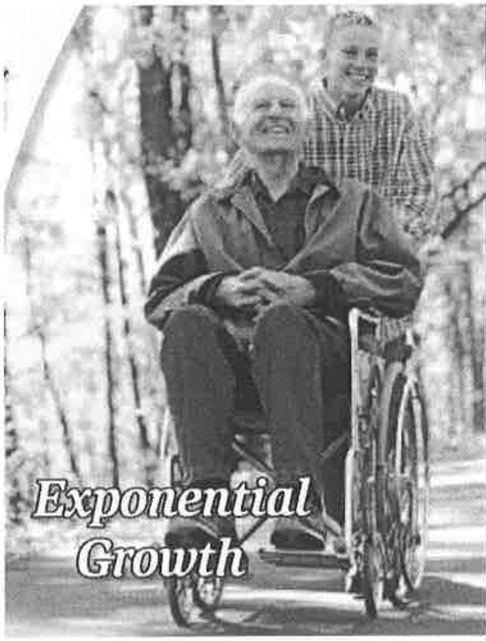
Key

Math 1 Name________________________

4-2 Exponential Growth Date________

- I can define an exponential function.
- I can convert a sequence into a recursive or explicit formula.



In the popular book and movie *Pay It Forward*, 12-year-old Trevor McKinney gets a challenging assignment from his social studies teacher.

Think of an idea for world change, and put it into practice!

Trevor came up with an idea that fascinated his mother, his teacher, and his classmates.

He suggested that he would do something really good for three people. Then when they ask how they can pay him back for the good deeds, he would tell them to "pay it forward"—each doing something good for three other people.

Trevor figured that those three people would do something good for a total of nine others. Those nine would do something good for 27 others, and so on. He was sure that before long there would be good things happening to billions of people all around the world.

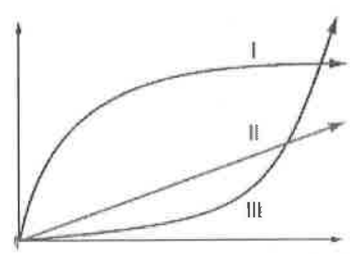
Think About This Situation

Continue Trevor's kind of Pay It Forward thinking.

a) How many people would receive a Pay It Forward good deed at each of the next several stages of the process?

b) What is your best guess about the number of people who would receive Pay It Forward good deeds at the tenth stage of the process?

c) Which of the graphs above do you think is most likely to represent the pattern by which the number of people receiving Pay It Forward good deeds increases as the process continues over time?



a. 3, 9, 27, 81, 243, 729, ...

b. 59,049 good deeds

c. Graph III → exponential!

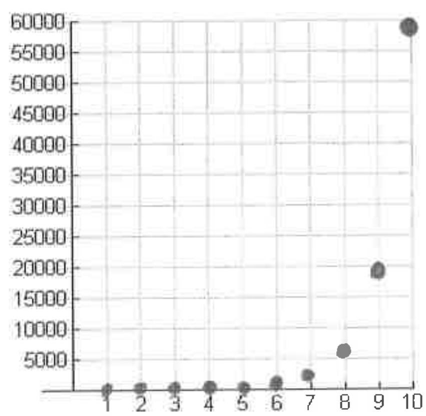
Counting in Tree Graphs

1. The number of good deeds in the Pay It Forward pattern can be represented by a *tree graph* that starts like this:

The vertices represent the people who receive and do good deeds. Each edge represents a good deed done by one person for another.

a. At the start of the Pay It Forward process, only one person does good deeds – for three new people. In the next stage, the three new people each do good things for three more new people. In the next stage, nine people each do good things for three more people, and so one, with no person receiving more than one good deed. Make a table that shows the number of good deeds for each stage. Then plot the data.

Stage	1	2	3	4	5	6	7	8	9	10
# of Good Deeds	3	9	27	81	243	729	2187	6561	19683	59049



b. The number of good deeds triples from one stage to the next.

How is this pattern shown in the plot?

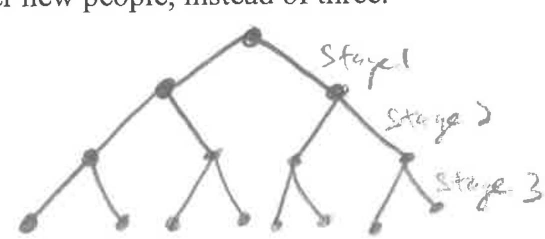
As x increases by 1, y increases by multiplying by 3. Increases slow at first then faster.

c. *For part (c), realize that the table in part (a) shows the number of good deeds done* ***at that stage****, not the total number of good deeds. For instance, at stage 3 there are 27 good deeds done, but the* ***TOTAL*** *number of good deeds done is 3 + 9 + 27 = 39.*

The number of stages needed before 25,000 ***total good deeds*** are done is 9.

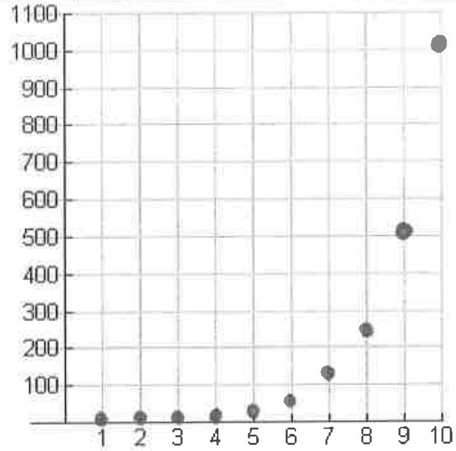
2. Consider now how the number of good deeds would grow if each person touched by the Pay It Forward process were to do good deeds for only two other new people, instead of three.

a. *Make a tree graph for* ***three*** *stages of this model below:*



b. Make a table showing the number of good deeds for the first 10 stages. Then label and plot.

Stage	1	2	3	4	5	6	7	8	9	10
# of Good Deeds	2	4	8	16	32	64	128	256	512	1024



c. The number of good deeds doubles from one stage to the next.

How this pattern is shown in the plot: Y doubles as x increases by 1. Increases slow at first, then faster.

d. The number of stages needed before 25,000 ***total*** good deeds are done is 14.

3.

a. <u>Recursive equation for Number (1):</u>

$$\begin{cases} a_1 = 3 \\ a_n = a_{n-1} \cdot 3 \end{cases}$$

<u>Recursive equation for Number (2):</u>

$$\begin{cases} a_1 = 2 \\ a_n = a_{n-1} \cdot 2 \end{cases}$$

c. Write a recursive equation that could be used to model a Pay It Forward process in which each person does good deeds for ***four*** other new people.

$$\begin{cases} a_1 = 4 \\ a_n = a_{n-1} \cdot 4 \end{cases}$$

5. Why would knowing an **explicit rule** be convenient?

It will allow you to easily find the # of good deeds done at any stage.

a. The explicit rule for the process in which each person does *three* good deeds for others is which of the following?

A. $N = 3x$ B. $N = x + 3$ C. $N = 3^x$ D. $N = 3x + 1$

How do you know?

Keep multiplying by 3 over & over $3 \cdot 3 \cdot 3 \cdot 3 \ldots$ x times, where x is the stage number.

b. Write an explicit rule that would show the number of good deeds at stage number *x* if each person in the process does good deeds for ***two*** others.

$N = 2^x$

c. Write an explicit rule that would show the number of good deeds at stage number *x* if each person in the process does good deeds for ***four*** others.

$N = 4^x$

Summary:

These patterns are no longer linear! They all increase at increasing rates.